6.6 Common Argument Forms and Fallacies

1. Common Valid Argument Forms: In the previous section (6.4), we learned how to determine whether or not an argument is valid using truth tables. There are certain forms of valid and invalid argument that are extremely common. If we memorize some of these common argument forms, it will save us time because we will be able to immediately recognize whether or not certain arguments are valid or invalid without having to draw out a truth table. Let’s begin:

1. Disjunctive Syllogism: The following argument is valid:

“The coin is either in my right hand or my left hand. It’s not in my right hand. So, it must be in my left hand.”

Let “R”=“The coin is in my right hand” and let “L”=“The coin is in my left hand”. The argument in symbolic form is this:

\[ R \lor L \]
\[ \sim R \]
\[ \therefore L \]

Any argument with the form just stated is valid. This form of argument is called a disjunctive syllogism. Basically, the argument gives you two options and says that, since one option is FALSE, the other option must be TRUE.

2. Pure Hypothetical Syllogism: The following argument is valid:

“If you hit the ball in on this turn, you’ll get a hole in one; and if you get a hole in one you’ll win the game. So, if you hit the ball in on this turn, you’ll win the game.”

Let “B”=“You hit the ball in on this turn”, “H”=“You get a hole in one”, and “W”=“you win the game”. The argument in symbolic form is this:

\[ B \supset H \]
\[ H \supset W \]
\[ \therefore B \supset W \]

Any argument with the form just stated is valid. This form of argument is called a pure hypothetical syllogism. Basically, the argument states a chain of reasons, where the first thing is connected to a second, and the second to a third, so the first is ultimately also connected to the third.
3. Modus Ponens (affirming the antecedent): The following argument is valid:

“If you have a driver’s license, then you must have taken the driver’s test. You do have a driver's license. So, you must have taken the driver’s test.”

Let “L”=“You have a driver’s license” and “T”=“You have taken the driver’s test”. The argument in symbolic form is this:

\[ \begin{array}{c}
L \land T \\
L \\
T
\end{array} \]

Any argument with the form just stated is valid. This form of argument is called by the Latin phrase, “modus ponens”. We’ll call it “affirming the antecedent”. Basically, the argument states that, given a first thing, a second thing is true. It then AFFIRMS that the first thing is true. So, the second thing must also be true.

4. Modus Tollens (denying the consequent): The following argument is valid:

“If you have a driver’s license, then you must have taken the driver’s test. You have not taken the driver’s test. So, you must not have a driver's license.”

Let “L”=“You have a driver’s license” and “T”=“You have taken the driver’s test”. The argument in symbolic form is this:

\[ \begin{array}{c}
L \land T \\
\neg T \\
\neg L
\end{array} \]

Any argument with the form just stated is valid. This form of argument is called by the Latin phrase, “modus tollens”. We’ll call it “denying the consequent”. Basically, the argument states that, given a first thing, a second thing is true. It then DENIES that the second thing is true. So, the first thing must also not be true.

2. Common Invalid Argument Forms: There are two very common INVALID argument forms which look a lot like modus ponens and modus tollens, but are mistaken.

1. Affirming the Consequent: The following argument is invalid:

“If you were standing out in the rain, then you would be wet now. You are wet now; so, you must have been standing out in the rain.”
Let “R”=“You were standing out in the rain” and let “W”=“You are wet now”. The argument in symbolic form is this:

\[
\begin{align*}
R & \supset W \\
W & \\
R & \\
\end{align*}
\]

Arguments with this form are generally invalid. This form of argument is called “affirming the consequent”. Basically, the argument states that, given a first thing, a second thing is true. It then AFFIRMS that the second thing is true, and concludes from this that the first thing must also be true.

But, this sort of inference is mistaken. For instance, just because you are wet does not guarantee that you were just standing out in the rain. Perhaps you just jumped into a pool, or into the shower. Or perhaps someone just poured a bucket of water on you. We cannot conclude for sure from the fact that you are soaking wet that you were outside in the rain.

2. Denying the Antecedent: The following argument is invalid:

“If you were standing out in the rain, then you would be wet now. You were not standing out in the rain. So, you must not be wet now.”

Let “R”=“You were standing out in the rain” and let “W”=“You are wet now”. The argument in symbolic form is this:

\[
\begin{align*}
R & \supset W \\
\sim R & \\
\sim W & \\
\end{align*}
\]

Arguments with this form are generally invalid. This form of argument is called “denying the antecedent”. Basically, the argument states that, given a first thing, a second thing is true. It then DENIES that the first thing is true, and then concludes from this that the second thing must also NOT be true.

But, this sort of inference is mistaken. For instance, just because you were not just standing in the rain does not guarantee that you are not soaking wet right now. Again, perhaps you just jumped into a pool, or into the shower. Or perhaps someone just poured a bucket of water on you. We cannot conclude for sure from the fact that you were not just outside in the rain that you are not soaking wet now.
Note: If an argument has one of the VALID argument forms, we CAN infer that it is valid for sure. But, if an argument has one of the INVALID argument forms, we CANNOT infer that it is invalid for sure. For instance, the following argument has the same form as the invalid “affirming the consequent” form. However, the following argument is VALID:

\[(A \lor B) \supset (A \land B)\]
\[A \land B\]
\[A \lor B\]

Let “p”=“(A \lor B)” and let “q”=“(A \land B)”. The argument in symbolic form is this:

\[p \supset q\]
\[q\]
\[p\]

Since this only occurs in more complicated arguments, however, you can ignore this sort of exception in this course.

3. Common Valid Dilemma Forms: There are two more valid argument forms. These come in the form of DILEMMAS.

1. Constructive Dilemma: The following argument is valid:

“If you take Logic, you will have to do a lot of homework; but, if you take Ethics, you will have to write a lot of papers. Since you must take either Logic or Ethics, you will either have to do a lot of homework or write a lot of papers.”

Let “L”=“You take Logic”, “H”=“You will do a lot of homework”, “E”=“You take Ethics”, and “P”=“You will write a lot of papers”. The argument in symbolic form is this:

\[(L \lor H) \land (E \lor P)\]
\[L \lor E\]
\[H \lor P\]

Any argument with the form just stated is valid. This form of argument is called a “constructive dilemma”. Basically, the argument states that two conditionals are true, and that either the antecedent of one or the other must be true; and, because one of the two antecedents must be true, it follows that one of the two consequents must also be true.
2. Destructive Dilemma: The following argument is valid:

“If you take Logic, you will have to do a lot of homework; but, if you take Ethics, you will have to write a lot of papers. Since you either don’t want to do a lot of homework or you don’t want to write a lot of papers, you should either not take Logic or not take Ethics.”

Let “L”=“You take Logic”, “H”=“You will do a lot of homework”, “E”=“You take Ethics”, and “P”=“You will write a lot of papers”. The argument in symbolic form is this:

\[(L \& H) \& (E \& P)\]
\[\sim H \vee \sim P\]
\[\sim L \vee \sim E\]

Any argument with the form just stated is valid. This form of argument is called a “destructive dilemma”. Basically, the argument states that two conditionals are true, and that either the consequent of one or the other must be true; and, because one of the two consequents must be false, it follows that one of the two antecedents must also be false.

4. Refuting Dilemmas: Remember that a successful argument must be both VALID and SOUND (that is, they must have a valid argument form, and all of the premises must be true). Since both of the dilemma forms we just looked at are VALID, if we want to reject the conclusion of a valid dilemma, we must show that they are not SOUND. In short, if we want to refute a valid dilemma, we must show that one of the premises is false. We can do this in one of two ways:

(1) Grasp the dilemma by the horns: The two forms of dilemma that we just looked at have two premises; the first is a conjunction, and the second is a disjunction. “Grasping the dilemma by the horns” involves rejecting the conjunction premise. To do this, we must show that one of the conjuncts is false. Let’s take the argument above about taking Ethics and Logic:

“If you take Logic, you will have to do a lot of homework; but, if you take Ethics, you will have to write a lot of papers. Since you must take either Logic or Ethics, you will either have to do a lot of homework or write a lot of papers.”

Perhaps some Logic professors do NOT assign a lot of homework; or perhaps some Ethics professors do NOT assign a lot of papers. In the first case, the conjunct “If you take Logic, then you will have to do a lot of homework” would be false. In the second case, the conjunct, “If you take Ethics, then you will have to write a lot of papers” would be false. So, if we can prove either of these things, we have refuted the dilemma.
Let’s look at another dilemma:

“If we feed starving children, then the population will increase too much; but if we don’t feed the starving children, then we allow horrible suffering to occur. Since we must either feed or not feed the starving children, either the population will increase too much, or we must allow horrible suffering to occur.”

Let “F”=“We feed starving children”, “P”=“Population increases too much”, and “S”=“We allow horrible suffering to occur”. The argument in symbolic form is this:

\[(F \lor P) \land (\neg F \lor S)\]
\[F \lor \neg F\]
\[\therefore P \lor S\]

This is a valid constructive dilemma. But, are both of the conditional conjuncts in the first premise TRUE? We might argue that the first conjunct is false (“If we feed starving children, then the population will increase too much”). Feeding starving people would not NECESSARILY result in a population explosion. For instance, perhaps at the same time as feeding them, we could teach them about procreation, and abstinence, or birth control, as well as the dangers of overpopulation.

(2) **Going between the horns:** The other way to refute a dilemma is to reject the disjunctive premise. To do this, we must show that the two options given in the second (disjunctive) premise are not the only two options available. Let’s look at the dilemma about Logic and Ethics first:

“If you take Logic, you will have to do a lot of homework; but, if you take Ethics, you will have to write a lot of papers. Since you must take either Logic or Ethics, you will either have to do a lot of homework or write a lot of papers.”

The disjunctive premise that we want to reject is this: “You must take either Logic or Ethics.” Perhaps this isn’t true. There might be some other way to fulfill your curriculum requirements. If there IS another way, then Logic and Ethics are not the only two options. In that case, we’ve “gone between the horns” of the dilemma and proved that the argument is unsound. Let’s look at the argument about famine relief:

“If we feed starving children, then the population will increase too much; but if we don’t feed the starving children, then we allow horrible suffering to occur. Since we must either feed or not feed the starving children, either the population will increase too much, or we must allow horrible suffering to occur.”
The disjunctive premise to be rejected here is that "We must either feed the starving children or not feed them." This premise has the following form:

\[ F \lor \neg F \]

Unfortunately, this is a **tautology** and so CANNOT be false. Whenever the disjunctive premise has the form “\( A \lor \neg A \)”, then there is no way for it to be false. It is automatically true because it exhausts ALL of the options. There are no other options. For instance, if I say, “Either there is milk in the fridge or there isn’t”, **there is no third alternative.** Either there is milk in there or there isn’t. Similarly, if I say, “I know your car is either red or not red”, there is NOT a third alternative. It’s either red or it isn’t. Whenever the disjunctive premise is a tautology, we cannot refute the dilemma by going between the horns. In that case, the ONLY way to refute the dilemma is by “grasping the horns of the dilemma”.

**5. Summary:** Here is a summary of the 6 valid forms, and 2 invalid forms that we’ve discussed so far:

<table>
<thead>
<tr>
<th>Form</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \lor q )</td>
<td>disjunctive</td>
</tr>
<tr>
<td>( \neg p ) \lor q</td>
<td></td>
</tr>
<tr>
<td>( p \lor q \lor r \lor \neg r )</td>
<td></td>
</tr>
<tr>
<td>( p \lor q \lor \neg r \lor r )</td>
<td></td>
</tr>
<tr>
<td>( p \lor q \lor \neg r \lor \neg q )</td>
<td></td>
</tr>
<tr>
<td>( p \lor q \lor \neg r \lor \neg q \lor r \lor \neg r )</td>
<td></td>
</tr>
</tbody>
</table>

Any argument having either of the following forms is invalid:

<table>
<thead>
<tr>
<th>Form</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \lor q \lor \neg r \lor r )</td>
<td></td>
</tr>
<tr>
<td>( p \lor q \lor \neg r \lor \neg q )</td>
<td></td>
</tr>
<tr>
<td>( p \lor q \lor \neg r \lor \neg q \lor r \lor \neg r )</td>
<td></td>
</tr>
</tbody>
</table>

**5. Some Rules to Remember:** The argument you are given may not always be in EXACTLY the right form. Here are some simple rules to remember:

(1) **You are allowed to change the order of the premises.**

For instance, the following is a valid disjunctive syllogism:
“The coin is not in my right hand. The coin is either in my right hand or my left hand. So, it must be in my left hand.”

The argument in symbolic form is this:

\[
\begin{align*}
\sim R \\
R \lor L \\
\hline
L
\end{align*}
\]

Technically, the premises are not in the right order. But, it is ok to swap them, so that they occur in the correct order, like this:

\[
\begin{align*}
R \lor L \\
\sim R \\
\hline
L
\end{align*}
\]

(2) **Negated letters can be substituted for “p” and “q”.**

For instance, above, I said that the following argument about famine relief was a valid constructive dilemma:

\[
\begin{align*}
(F \land P) \land (~F \land S) \\
F \lor \sim F \\
\hline
P \lor S
\end{align*}
\]

But, it doesn’t EXACTLY have the same form as a constructive dilemma, because the second disjunct is negated. The **EXACT** correct form of a constructive dilemma is the following:

\[
\begin{align*}
(p \land q) \land (r \land s) \\
p \lor r \\
\hline
q \lor s
\end{align*}
\]

But, if we just substitute “r” for “\(\sim F\)” we DO get the correct form. This is perfectly acceptable.

(3) **“A” is equivalent to “\(~\sim A\)”**

Basically, a double negative is positive. If I tell you that I don’t NOT teach Logic, this means that I DO teach Logic. Sometimes, you will need to replace letters like “A” with double negatives like “\(~\sim A\)” in order to get the correct form. For instance, if someone says the following:
“I'll give ya a hint: Either Harry is not married, or he’s cheating. And by the way he IS married.”

We can conclude from this that Harry is cheating, because this IS a valid disjunctive syllogism. Let “M”=“Harry is married” and let “C”=“Harry is cheating”. The argument has the following form:

\[
\begin{array}{c}
\sim M \lor C \\
M \\
C \\
\end{array}
\]

But, the correct argument form for a disjunctive syllogism is this:

\[
\begin{array}{c}
p \lor q \\
\sim p \\
q \\
\end{array}
\]

It’s not quite right. But if we put TWO negations in front of the second premise (“M”), we DO get the correct form. Like this:

\[
\begin{array}{c}
\sim M \lor C \\
\sim \sim M \\
C \\
\end{array}
\]

This is a perfectly acceptable way to change an argument.

(4) “A \lor B” is equivalent to “B \lor A”

When we learned about disjunction, we learned that it does not matter if you swap the order of the disjuncts. For instance, imagine that above we had given this argument:

“The coin is either in my left hand or my right hand. It’s not in my right hand. So, it must be in my left hand.”

Let “R”=“The coin is in my right hand” and let “L”=“The coin is in my left hand”. The argument in symbolic form is this:

\[
\begin{array}{c}
L \lor R \\
\sim R \\
L \\
\end{array}
\]
To be very technical about it, the correct argument form is SUPPOSED to be when you negate the LEFT disjunct, not the RIGHT one. But, this is fixed easily enough. We can simply swap the order of the disjuncts in the first premise, like this:

\[
\begin{align*}
R \lor L \\
\sim R \\
\hline
L
\end{align*}
\]

Making this change is perfectly acceptable.

*Note: Do homework for section 6.6 at this time.*