From: Hurley (ed.), A Concise Introduction to Logic, 11th edition

1.5 Argument Forms: Proving Invalidity

This section explores the idea that the validity of a deductive argument is determined by the **argument form.** This idea was suggested in the arguments about wines and beverages presented in Table 1.1 in the previous section. All the arguments in the valid column have the same form, and all the arguments in the invalid column have the same form.

1

Yet, in the exercises at the end of that section we saw many cases of valid deductive arguments that did not have any recognizable form. How can we reconcile this fact with the claim that validity is determined by form? The answer is that these arguments are incomplete, so the form is not explicit. But once such arguments are completed and correctly phrased (which we address later in this book), the form becomes apparent. For example, consider the following valid argument:

Geese are migratory waterfowl, so they fly south for the winter.

This argument is missing a premise:

Migratory waterfowl fly south for the winter.

The argument can now be rephrased to make its form apparent:

All geese are migratory waterfowl.
All migratory waterfowl are birds that fly south for the winter.
Therefore, all geese are birds that fly south for the winter.

The form of the argument is

All *A* are *B*.
All *B* are *C*.
____
All *A* are *C*.

This form is valid, and it captures the reasoning process of the argument. If we assume that the *A*s (whatever they might be) are included in the *B*s, and that the *B*s (whatever they might be) are included in the *C*s, then the *A*s must necessarily be included in the *C*s. This necessary relationship between the *A*s, *B*s, and *C*s is what makes the argument valid. This is what we mean when we say that the validity of a deductive argument is determined by its form.

Since validity is determined by form, it follows that any argument that has this valid form is a valid argument. Thus, we might substitute "daisies" for *A*, "flowers" for *B*, and "plants" for *C* and obtain the following valid argument:

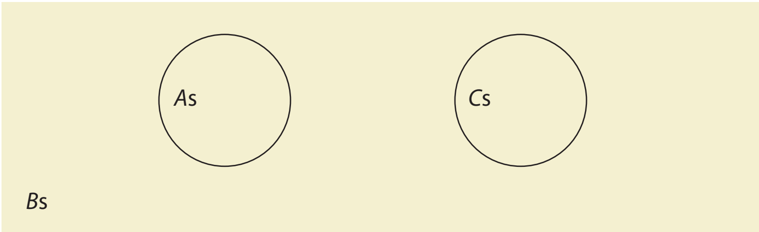
All daisies are flowers.
All flowers are plants.
Therefore, all daisies are plants.

Any argument such as this that is produced by uniformly substituting terms or statements in place of the letters in an argument form is called a **substitution instance** of that form.

Let us now consider an invalid argument form:

All *A* are *B*.
All *C* are *B*.
____
All *A* are C.

In this argument form, if we assume that the *A*s are in the *B*s and that the *C*s are in the *B*s, it does not *necessarily* follow that the *A*s are in the *C*s. It would not follow if the *A*s were in one part of the *B*s and the *C*s were in another part, as the following diagram illustrates:



This diagram suggests that we can prove the form invalid if we can find a substitution instance having actually true premises and an actually false conclusion. In such a substitution instance the *A*s and the *C*s would be separated from each other, but they would both be included in the *B*s. If we substitute "cats" for *A*, "animals" for *B*, and "dogs" for *C*, we have such a substitution instance:

All *A* are *B*.	All cats are animals.	True
All *C* are *B*.	All dogs are animals.	True
All *A* are *C*.	Therefore, all cats are dogs.	False

This substitution instance proves the form invalid, because it provides a concrete example of a case where the *A*s are in the *B*s, the *C*s are in the *B*s, but the *A*s are *not* in the *C*s.

Now, since the form is invalid, can we say that any argument that has this form is invalid? Unfortunately, the situation with invalid forms is not quite as simple as it is with valid forms. Every substitution instance of a valid form is a valid argument, but it is not the case that every substitution instance of an invalid form is an invalid argument. The reason is that some substitution instances of invalid forms are also substitution instances of valid forms.* However, we can say that any substitution instance of an invalid form is an invalid argument *provided* that it is not a substitution instance of any valid form. Thus, we will say that an argument actually *has* an invalid form if it is a substitution instance of that form and it is not a substitution instance of any valid form.

The fact that some substitution instances of invalid forms are also substitution instances of valid forms means simply that we must exercise caution in identifying the form of an argument. However, cases of ordinary language arguments that can be interpreted as substitution instances of both valid and invalid forms are so rare that this book chooses to ignore them. With this in mind, consider the following argument:

*For example, the following valid argument is a substitution instance of the invalid form we have been discussing:

All bachelors are persons.
All unmarried men are persons.
Therefore, all bachelors are unmarried men.

However, because "bachelors" is equivalent in meaning to "unmarried men," the argument is also a substitution instance of this valid form:

All *A* are *B*.
All *A* are *B*.
All *A* are *A*.

1

All romantic novels are literary pieces.
All works of fiction are literary pieces.
Therefore, all romantic novels are works of fiction.

This argument clearly has the invalid form just discussed. This invalid form captures the reasoning process of the argument, which is obviously defective. Therefore, the argument is invalid, and it is invalid precisely because it has an invalid form.

Counterexample Method

A substitution instance having true premises and a false conclusion (like the cats-and-dogs example just constructed) is called a counterexample, and the method we have just used to prove the romantic-novels argument invalid is called the **counterexample method.** It consists of isolating the form of an argument and then constructing a substitution instance having true premises and a false conclusion. This proves the form invalid, which in turn proves the argument invalid. The counterexample method can be used to prove the invalidity of any invalid argument, but it cannot prove the validity of any valid argument. Thus, before the method is applied to an argument, the argument must be known or suspected to be invalid in the first place. Let us apply the counterexample method to the following invalid categorical syllogism:

Since some employees are not social climbers and all vice presidents are employees, we may conclude that some vice presidents are not social climbers.

This argument is invalid because the employees who are not social climbers might not be vice presidents. Accordingly, we can *prove* the argument invalid by constructing a substitution instance having true premises and a false conclusion. We begin by isolating the form of the argument:

Some *E* are not *S*.
All *V* are *E*.
Some *V* are not *S*.

Next, we select three terms to substitute in place of the letters that will make the premises true and the conclusion false. The following selection will work:

E = animals
S = mammals
V = dogs

The resulting substitution instance is this:

Some animals are not mammals.
All dogs are animals.
Therefore, some dogs are not mammals.

The substitution instance has true premises and a false conclusion and is therefore, by definition, invalid. Because the substitution instance is invalid, the form is invalid, and therefore the original argument is invalid.

In applying the counterexample method to categorical syllogisms, it is useful to keep in mind the following set of terms: "cats," "dogs," "mammals," "fish," and "animals." Most invalid syllogisms can be proven invalid by strategically selecting three of these terms and using them to construct a counterexample. Because everyone agrees about these terms, everyone will agree about the truth or falsity of the premises and conclusion of the counterexample. Also, in constructing the counterexample, it often helps to begin with the conclusion. First, select two terms that yield a false conclusion, and then select a third term that yields true premises. Another point to keep in mind is that the word "some" in logic always means "at least one." For example, the statement "Some dogs are animals" means "At least one dog is an animal"—which is true. Also note that this statement does not imply that some dogs are not animals.

Not all deductive arguments, of course, are categorical syllogisms. Consider, for example, the following hypothetical syllogism:

> If the government imposes import restrictions, the price of automobiles will rise. Therefore, since the government will not impose import restrictions, it follows that the price of automobiles will not rise.

This argument is invalid because the price of automobiles might rise even though import restrictions are not imposed. It has the following form:

If *G*, then *P*.
Not *G*.
———
Not *P*.

This form differs from the previous one in that its letters stand for complete statements. *G*, for example, stands for "The government imposes import restrictions." If we make the substitution

G = Abraham Lincoln committed suicide.
P = Abraham Lincoln is dead.

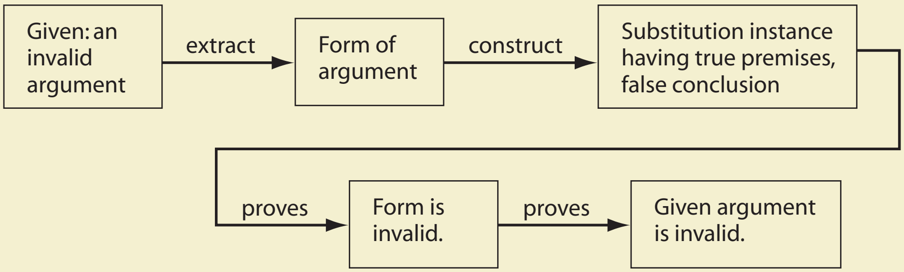
we obtain the following substitution instance:

> If Abraham Lincoln committed suicide, then Abraham Lincoln is dead.
> Abraham Lincoln did not commit suicide.
> Therefore, Abraham Lincoln is not dead.

Since the premises are true and the conclusion false, the substitution instance is clearly invalid. Therefore, the form is invalid, and this proves the original argument invalid.

When applying the counterexample method to an argument having a conditional statement as a premise (such as the one just discussed), it is recommended that the statement substituted in place of the conditional statement express some kind of necessary connection. In the Lincoln example, the first premise asserts the necessary connection between suicide and death. There can be no doubt about the truth of such a statement. Furthermore, if it should turn out that the conclusion is a conditional statement, note that one sure way of producing a false conditional statement is by joining a true antecedent with a false consequent. For example, the conditional statement "If Lassie is a dog, then Lassie is a cat" is clearly false.

Counterexample method



Being able to identify the form of an argument with ease requires a familiarity with the basic deductive argument forms. The first task consists in distinguishing the premises from the conclusion. Always write the premises first and the conclusion last. The second task involves distinguishing what we may call "form words" from "content words." To reduce an argument to its form, leave the form words as they are, and replace the content words with letters. For categorical syllogisms, the words "all," "no," "some," "are," and "not" are form words, and for hypothetical syllogisms the words "if," "then," and "not" are form words. Additional form words for other types of arguments are "either," "or," "both," and "and." For various kinds of hybrid arguments, a more intuitive approach may be needed. Here is an example:

> All movie stars are actors who are famous, because all movie stars who are famous are actors.

If we replace "movie stars," "actors," and "famous" with the letters *M*, *A*, and *F*, this argument has the following form:

> All *M* who are *F* are *A*.
> All *M* are *A* who are *F*.

Here is one possible substitution instance for this form:

> All humans who are fathers are men.
> Therefore, all humans are men who are fathers.

Because the premise is true and the conclusion false, the form is invalid and so is the original argument.

Using the counterexample method to prove arguments invalid requires a little ingenuity because there is no rule that will automatically produce the required term or statement to be substituted into the form. Any term or statement will work, of course, provided that it yields a substitution instance that has premises that are indisputably true and a conclusion that is indisputably false. Ideally, the truth value of these statements should be known to the average individual; otherwise, the substitution instance cannot be depended on to prove anything. If, for example, *P* in the earlier hypothetical syllogism had been replaced by the statement "George Wilson is dead," the substitution instance would be useless, because nobody knows whether this statement is true or false.

The counterexample method is useful only for proving invalidity, because the only arrangement of truth and falsity that proves anything is true premises and false conclusion. If a substitution instance is produced having true premises and a true conclusion, it does *not* prove that the argument is valid. Furthermore, the method is useful only for deductive arguments because the strength and weakness of inductive arguments is only partially dependent on the form of the argument. Accordingly, no method that relates exclusively to the form of an inductive argument can be used to prove the argument weak.