30 The sorites paradox

Suppose (if you have to suppose) that you have a full head of hair. That means that you probably have around 100,000 individual hairs. Now pull one of them out. Does that make you bald? Of course not. A single hair doesn't make any difference. 99,999 hairs still make a full head of hair.

Indeed, we would surely all agree that, if you are not bald, removing just one hair could never make you bald. And yet, if you pull out another hair, and another, and another . . . Eventually, if you carry on long enough, you will have none left and you will indubitably be bald. So you apparently move from a state of unquestionable non-baldness to a state of unquestionable baldness by taking a series of steps that can never on their own have that effect. So when did the change come about?

This is a version of a famous puzzle, usually attributed to the ancient Greek logician Eubulides of Miletus, known as the sorites paradox. 'Sorites' comes from the Greek word *soros*, meaning a 'heap', as the original formulation of the puzzle features a heap of sand. Expressed in terms of addition (of sand grains) rather than subtraction (of hairs), the argument looks like this:

1 grain of sand does not make a heap.
If 1 grain does not make a heap, then 2 grains do not.
If 2 grains do not make a heap, then 3 grains do not.
[and so on until . . .]

timeline

c.350BC Forms of argument

c.300BC The sorites paradox

If 99,999 grains do not make a heap, then 100,000 grains do not.
So 100,000 grains of sand do not make a heap.

But everybody would surely baulk at this conclusion. So what can have gone wrong?

Problems of vagueness Faced with an unpalatable conclusion of this kind, it is necessary to track back over the argument by which it has been reached. There must be something wrong with the premises on which the argument is based or some error in the reasoning. In fact, in spite of its great antiquity, there is still no clear consensus on how best to tackle this paradox, and various approaches have been taken.

One way out of the paradox is to insist, as some have done, that there is a point at which adding a grain of sand makes a difference; that there is a precise number of grains of sand that marks the boundary between a heap and a non-heap. If there is such a boundary, clearly we do not know where it is, and any proposed dividing line sounds hopelessly arbitrary: do 1001

Terminal logic

Smokers with ostrich-like tendencies are often susceptible to the kind of faulty reasoning that underlies the sorites paradox. The smoker reasons, not implausibly, that 'the next one won't kill me'. Having established this, she moves by an effortless soritic progression to the claim that 'the one after the next one won't kill me'. And so on, but sadly not ad infinitum. The probable truth that no single cigarette will kill you (though the sum of cigarettes smoked very likely will) represents a pyrrhic victory for the late smoker.

Fuzzy logic

Traditional logic is bivalent, which means that only two truth values are allowed: every proposition must be either true or false. But the inherent vagueness of many terms, apparent in the sorites paradox, suggests that this requirement is too rigid if logic is to encompass the full scope and complexity of natural language.

Fuzzy logic has been developed, initially by the computer scientist Lofti Zadeh, to allow for imprecision and degrees of truth. Truth is presented as a continuum between true (1) and false (0). So, for instance, a particular proposition that is 'partly true' or 'more or less true' might be represented as true to degree 0.8 and false to degree 0.2. Fuzzy logic has been particularly important in AI (artificial intelligence) research, where 'intelligent' control systems need to be responsive to the imprecisions and nuances of natural language.

'As complexity rises, precise statements lose meaning and meaningful statements lose precision.'

Lofti Zadeh, 1965

grains, say, make a heap, but not 999? This really is a big slap in the face for common sense and our shared intuitions.

More promising is to take a closer look at a major assumption underlying the argument: the idea that the process of construction by which a non-heap becomes a heap can be fully and reductively analysed into a series of discrete grain additions. Clearly there are a number of such discrete steps, but equally clearly it seems that these steps are not fully constitutive of the overall process of heap-building.

This faulty analysis fails to recognize that the transition from non-heap to heap is a continuum, and hence that there is no precise point at which the change can be said to occur (for similar problems concerning vagueness, see page 87). This in turn tells us something about the whole class of terms to which the sorites paradox can be applied: not only heap and bald, but also tall, big, rich, fat and countless others. All of these terms are

«There are no whole truths; all truths are half-truths. It is trying to treat them as whole truths that plays the devil.»

Alfred North Whitehead, 1953

essentially vague, with no clear dividing line separating them from their opposites – short, small, poor, thin, etc.

One important consequence of this is that there are always borderline cases where the terms do not clearly apply. So, for instance, while there may be some people who are clearly bald and others who are clearly not, there are many in between who might, according to context and circumstances, be designated as one or the other. This inherent vagueness means that it is not always appropriate to say of a sentence such as 'X is bald' that it is (unequivocally) true or false; rather, there are degrees of truth. This at once creates a tension between these vague terms that occur in natural language and classical logic, which is *bivalent* (meaning that every proposition must be either true or false).

The concept of vagueness suggests that classical logic must be overhauled if it is to fully capture the nuances of natural language. For this reason there has been a move towards the development of fuzzy and other multivalued logics (see box).

the condensed idea

How many grains make a heap?