6

The Identity of Indiscernibles

Max Black

A: The principle of the Identity of Indiscernibles seems to me obviously true. And I don't see how we are going to define identity or establish the connection between mathematics and logic without using it.

B: It seems to me obviously false. And your troubles as a mathematical logician are beside the point. If the principle is false, you have no right to use it.

A: You simply *say* it's false – and even if you said so three times, that wouldn't make it so.

B: Well, you haven't done anything more yourself than assert the principle to be true. As Bradley once said, 'assertion can demand no more than counter-assertion; and what is affirmed on the one side, we on the other can simply deny.'

A: How will this do for an argument? If two things, *a* and *b*, are given, the first has the property of being identical with *a*. Now *b* cannot have this property, for else *b* would be *a*, and we should have only one thing, not two as assumed. Hence *a* has at least one property, which *b* does not have, that is to say the property of being identical with *a*.

B: This is a roundabout way of saying nothing, for '*a* has the property of being identical with *a*' means no more than '*a* is *a*'. When you begin to say '*a* is . . .' I am supposed to know what thing you are referring to as '*a*', and I expect to be told something about that thing. But when you end the sentence with the words '. . . is *a*', I am left still waiting. The sentence '*a* is *a*' is a useless tautology.

A: Are you as scornful about difference as about identity? For *a* also has, and *b* does not have, the property of being different from *b*. This is a second property that the one thing has but not the other.

B: All you are saying is that *b* is different from *a*. I think the form of words '*a* is different from *b*' does have the advantage over '*a* is *a*' that it might be used to give information. I might learn from hearing it used that '*a*' and '*b*' were applied to different things. But this is not what you want to say, since you are trying to use the names, not mention them. When I already know what '*a*' and '*b*' stand for, '*a* is different from *b*' tells me nothing. It, too, is a useless tautology.

A: I wouldn't have expected you to treat 'tautology' as a term of abuse. Tautology or not, the sentence has a philosophical use. It expresses the necessary truth that different things have at least one property not in common. Thus different things must be discernible; and hence, by contraposition, indiscernible things must be identical. Q.E.D.

B: Why obscure matters by this old-fashioned language? By 'indiscernible' I suppose you mean the same as 'having all properties in common' Do you claim to have proved that two things having all their properties in common are identical?

A: Exactly.

B: Then this is a poor way of stating your conclusion. If *a* and *b* are identical, there is just one thing having the two names '*a*' and '*b*'; and in that case it is absurd to say that *a* and *b* are two. Conversely, once you have supposed there are *two*

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things having all their properties in common, you can't without contradicting yourself say that *they* are 'identical'.

A: I can't believe you were really misled. I simply meant to say it is logically impossible for two things to have all their properties in common. I showed that *a* must have at least two properties – the property of being identical with *a* and the property of being different from *b* – neither of which can be a property of *b*. Doesn't this prove the principle of identity of indiscernibles?

B: Perhaps you have proved something. If so, the nature of your proof should show us exactly what you have proved. If you want to call 'being identical with *a*' a 'property' I suppose I can't prevent you. But you must then accept the consequences of this way of talking. All you mean when you say '*a* has the property of being identical with *a*' is that *a* is *a*. And all you mean when you say '*b* does not have the property of being identical with *a*' is that *b* is not *a*. So what you have 'proved' is that *a* is *a* and *b* is not *a*; that is to say, *b* and *a* are different. Similarly, when you said that *a*, but not *b*, had the property of being different from *b*, you were simply saying that *a* and *b* were different. In fact you are merely redescribing the hypothesis that *a* and *b* are different by calling it a case of 'difference of properties'. Drop the misleading description and your famous principle reduces to the truism that different things are different. How true! And how uninteresting!

A: Well, the properties of identity and difference may be uninteresting, but they *are* properties. If I had shown that grass was green, I suppose you would say I hadn't shown that grass was coloured.

B: You certainly would not have shown that grass had any colour *other than* green.

A: What it comes to is that you object to the conclusion of my argument *following* from the premiss that *a* and *b* are different.

B: No, I object to the triviality of the conclusion. If you want to have an interesting principle to defend, you must interpret 'property' more narrowly – enough so, at any rate, for 'identity' and 'difference' not to count as properties.

A: Your notion of an interesting principle seems to be one which I shall have difficulty in establishing. Will you at least allow me to include among 'properties' what are sometimes called 'relational characteristics' – like *being married to Caesar* or *being at a distance from London*?

B: Why not? If you are going to defend the principle, it is for you to decide what version you wish to defend.

A: In that case, I don't need to count identity and difference as properties. Here is a different argument that seems to me quite conclusive. The only way we can discover that two different things exist is by finding out that one has a quality not possessed by the other or else that one has a relational characteristic that the other hasn't.

If *both* are blue and hard and sweet and so on, and have the same shape and dimensions and are in the same relations to everything in the universe, it is logically impossible to tell them apart. The supposition that in such a case there might really be two things would be unverifiable *in principle*. Hence it would be meaningless.

B: You are going too fast for me.

A: Think of it this way. If the principle were false, the fact that I can see only two of your hands would be no proof that you had just two. And even if every conceivable test agreed with the supposition that you had two hands, you might all the time have three, four, or any number. You might have nine hands, different from one another and all indistinguishable from your left hand, and nine more all different from each other but indistinguishable from your right hand. And even if you really did have just two hands, and no more, neither you nor I nor anybody else could ever know that fact. This is too much for me to swallow. This is the kind of absurdity you get into, as soon as you abandon verifiability as a test of meaning.

B: Far be it from me to abandon your sacred cow. Before I give you a direct answer, let me try to describe a counter-example.

Isn't it logically possible that the universe should have contained nothing but two exactly similar spheres? We might suppose that each was made of chemically pure iron, had a diameter of one mile, that they had the same temperature, colour, and so on, and that nothing else existed. Then every quality and relational characteristic of the one would also be a property of the other. Now if what I am describing is logically possible, it is not impossible for two things to have all their properties in common. This seems to me to *refute* the Principle.

A: Your supposition, I repeat, isn't verifiable and therefore can't be regarded as meaningful. But supposing you *have* described a possible world, I still don't see that you have refuted the principle. Consider one of the spheres, *a*, . . .

B: How can I, since there is no way of telling them apart? *Which* one do you want me to consider?

A: This is very foolish. I mean either of the two spheres, leaving you to decide which one you wished to consider. If I were to say to you 'Take any book off the shelf', it would be foolish on your part to reply 'Which?'

B: It's a poor analogy. I know how to take a book off a shelf, but I don't know how to identify one of two spheres supposed to be alone in space and so symmetrically placed with respect to each other that neither has any quality or character the other does not also have.

A: All of which goes to show as I said before, the unverifiability of your supposition. Can't you imagine that one sphere has been designated as '*a*'?

B: I can imagine only what is logically possible. Now it is logically possible that somebody should enter the universe I have described, see one of the spheres on his left hand and proceed to call it '*a*'. I can imagine that all right, if that's enough to satisfy you.

A: Very well, now let me try to finish what I began to say about *a*...

B: I still can't let you, because you, in your present situation, have no right to talk about *a*. All I have conceded is that if something were to happen to introduce a change into my universe, so that an observer entered and could see the two spheres, one of them could then have a name. But this would be a different supposition from the one I wanted to consider. My spheres don't yet have names. If an observer were to enter the scene, he could perhaps put a red mark on one of the spheres. You might just as well say 'By "*a*" I mean the sphere which would be the first to be marked by a red mark if anyone were to arrive and were to proceed to make a red mark!' You might just as well ask me to consider the first daisy in my lawn that would be picked by a child, if a child were to come along and do the picking. This doesn't now distinguish any daisy from the others. You are just pretending to use a name.

A: And I think you are just pretending not to understand me. All I am asking you to do is to think of one of your spheres, no matter which, so that I may go on to say something about it when you give me a chance.

B: You talk as if naming an object and then thinking about it were the easiest thing in the world. But it isn't so easy. Suppose I tell you to name any spider in my garden: if you can catch one first or describe one uniquely, you can name it easily enough. But you can't pick one out, let alone 'name' it, by just thinking. You remind me of the mathematicians who thought that talking about an Axiom of Choice would really allow them to choose a single member of a collection when they had no criterion of choice.

A: At this rate you will never give me a chance to say anything. Let me try to make my point without using names. Each of the spheres will surely differ from the other in being at some distance from that other one, but at no distance from itself – that is to say, it will bear at least one relation to itself – *being at no distance from*, or *being in the same place as* – that it does not bear to the other. And this will serve to distinguish it from the other.

B: Not at all. *Each* will have the relational characteristic *being at a distance of two miles*, say, *from the centre of a sphere one mile in diameter*, etc. And each will have the relational characteristic (if you want to call it that) of *being in the same place as itself.* The two are alike in this respect as in all others.

A: But look here. Each sphere occupies a different place; and this at least will distinguish them from one another.

B: This sounds as if you thought the places had some independent existence, though I don't suppose you really think so. To say the spheres are in 'different places' is just to say that there is a distance between the two spheres; and we have already seen that that will not serve to distinguish them. Each is at a distance – indeed the same distance – from the other.

A: When I said they were at different places, I didn't mean simply that they were at a distance from one another. That one sphere is in a certain place does not entail the existence of any *other* sphere. So to say that one sphere is in its place, and the other in its place, and then to add that these places are different seems to me different from saying the spheres are at a distance from one another.

B: What does it mean to say 'a sphere is in its place'? Nothing at all, so far as I can see. Where else could it be? *All* you are saying is that the spheres are in different places.

A: Then my retort is, What does it mean to say 'Two spheres are in different places'? Or, as you so neatly put it, 'Where else could they be?'

B: You have a point. What I should have said was that your assertion that the spheres occupied different places said nothing at all, unless you were drawing attention to the necessary truth that different physical objects must be in different places. Now if two spheres must be in different places, as indeed they must, to say that the spheres occupy different places is to say no more than they are two spheres.

A: This is like a point you made before. You won't allow me to deduce anything from the supposition that there are two spheres.
B: Let me put it another way. In the two-sphere universe, the only reason for saying that the places occupied were different would be that different things occupied them. So in order to show the places were different, you would first have to show, in some other way, that the spheres were different. You will never be able to distinguish the spheres by means of the places they occupy.
A: A minute ago, you were willing to allow that somebody might give your spheres different names. Will you let me suppose that some traveller has visited your monotonous 'universe' and has named one sphere 'Castor' and the other 'Pollux'?
B: All right – provided you don't try to use those names yourself.
A: Wouldn't the traveller, at least, have to recognize that *being at a distance of two miles from Castor* was not the same property as being at a distance of two miles *from Pollux*?
B: I don't see why. If he were to see that Castor and Pollux had exactly the same properties, he would see that 'being at a distance of two miles from Castor' meant exactly the same as 'being at a distance of two miles from Pollux'.
A: They couldn't mean the same. If they did, '*being at a distance of two miles from Castor and at the same time not being at a distance of two miles from Pollux*' would be a self-contradictory description. But plenty of bodies could answer to this description. Again, if the two expressions meant the same, anything which was two miles from Castor would have to be two miles from Pollux – which is clearly false. So the two expressions don't mean the same, and the two spheres have at least two properties not in common.
B: Which?
A: *Being at a distance of two miles from Castor* and *being at a distance of two miles from Pollux*.
B: But now you are *using* the words 'Castor' and 'Pollux' as if they really stood for something. They are just our old friends '*a*' and '*b*' in disguise.
A: You surely don't want to say that the arrival of the name-giving traveller creates spatial properties? Perhaps we can't name your spheres and therefore can't name the corresponding properties; but the properties must be there.
B: What can this mean? The traveller has not visited the spheres, and the spheres have no names – neither 'Castor', nor 'Pollux', nor '*a*', nor '*b*', nor any others. Yet you still want to say they have certain properties which cannot be referred to without using names for the spheres. You want to say 'the property of being at a distance from Castor', though it is logically impossible for you to talk in this way. You can't speak, but you won't be silent.
A: How eloquent, and how unconvincing! But since you seem to have convinced yourself, at least, perhaps you can explain another thing that bothers me: I don't see that you have a right to talk as you do about places or spatial relations in connection with your so-called universe. So long as we are talking about our own universe – *the* universe – I know what you mean by 'distance', 'diameter', 'place' and so on. But in what you want to call a universe, even though it contains only two objects, I don't see what such words could mean. So far as I can see, you are applying these spatial terms in their present usage to a hypothetical situation which contradicts the presuppositions of that usage.
B: What do you mean by 'presupposition'?
A: Well, you spoke of measured distances, for one thing. Now this presupposes some means of measurement. Hence your 'universe' must contain at least a third thing – a ruler or some other measuring device.
B: Are you claiming that a universe must have at least three things in it? What is the least number of things required to make a world?
A: No, all I am saying is that you cannot describe a configuration as *spatial* unless it includes at least three objects. This is part of the meaning of 'spatial' – and it is no more mysterious than saying you can't have a game of chess without there existing at least thirty-five things (thirty-two pieces, a chessboard, and two players).
B: If this is all that bothers you, I can easily provide for three or any number of things without changing the force of my counter-example. The important thing, for my purpose, was that the configuration of two spheres was symmetrical. So long as we preserve this feature of the imaginary universe, we can now allow any number of objects to be found in it.
A: You mean any *even* number of objects.
B: Quite right. Why not imagine a plane running clear through space, with everything that happens on one side of it always exactly duplicated at an equal distance in the other side.
A: A kind of cosmic mirror producing real images.
B: Yes, except that there wouldn't be any mirror! The point is that in *this* world we can imagine any

degree of complexity and change to occur. No reason to exclude rulers, compasses and weighing machines. No reason, for that matter, why the Battle of Waterloo shouldn't happen.

A: Twice over, you mean – with Napoleon surrendering later in two different places simultaneously!

B: Provided you wanted to call both of them 'Napoleon'.

A: So your point is that everything could be duplicated on the other side of the non-existent Looking Glass. I suppose whenever a man got married, his identical twin would be marrying the identical twin of the first man's fiancée?

B: Exactly.

A: Except that 'identical twins' wouldn't be *numerically* identical?

B: You seem to be agreeing with me.

A: Far from it. This is just a piece of gratuitous metaphysics. If the inhabitants of your world had enough sense to know what was sense and what wasn't, they would never suppose all the events in their world were duplicated. It would be much more sensible for them to regard the 'second' Napoleon as a mere mirror image – and similarly for all the other supposed 'duplicates'.

B: But they could walk through the 'mirror' and find water just as wet, sugar just as sweet, and grass just as green on the other side.

A: You don't understand me. They would not postulate 'another side'. A man looking at the 'mirror' would be seeing *himself*, not a duplicate. If he walked in a straight line toward the 'mirror', he would eventually find himself back at his starting point, not at a duplicate of his starting point. This would involve their having a different geometry from ours – but that would be preferable to the logician's nightmare of the reduplicated universe.

B: They might think so – until the twins really began to behave differently for the first time!

A: Now it's you who are tinkering with your supposition. You can't have your universe and change it too.

B: All right, I retract.

A: The more I think about your 'universe', the queerer it seems. What would happen when a man crossed your invisible 'mirror'? While he was actually crossing, his body would have to change shape, in order to preserve the symmetry. Would it gradually shrink to nothing and then expand again?

B: I confess I hadn't thought of that.

A: And here is something that explodes the whole notion. Would you say that one of the two Napoleons in your universe had his heart in the right place – literally, I mean?

B: Why, of course.

A: In that case his 'mirror-image' twin would have the heart on the opposite side of the body. One Napoleon would have his heart on the left of his body, and the other would have it on the right of his body.

B: It's a good point, though it would still make objects like spheres indistinguishable. But let me try again. Let me abandon the original idea of a *plane* of symmetry and suppose instead that we have only a *centre* of symmetry. I mean that everything that happened at any place would be exactly duplicated at a place an equal distance on the opposite side of the centre of symmetry. In short, the universe would be what the mathematicians call 'radially symmetrical'. And to avoid complications, we could suppose that the centre of symmetry itself was physically inaccessible, so that it would be impossible for any material body to pass through it. Now in *this* universe, identical twins would have to be either both right-handed or both left-handed.

A: Your universes are beginning to be as plentiful as blackberries. You are too ingenuous to see the force of my argument about verifiability. Can't you see that your supposed description of a universe in which everything has its 'identical twin' doesn't describe anything verifiably different from a corresponding universe without such duplication? This must be so, no matter what kind of symmetry your universe manifested.

B: You are assuming that in order to verify that there are two things of a certain kind, it must be possible to show that one has a property not possessed by the other. But this is not so. A pair of very close but similar magnetic poles produce a characteristic field of force which assures me that there are two poles, even if I have no way of examining them separately. The presence of two exactly similar stars at a great distance might be detected by some resultant gravitational effect or by optical interference – or in some such similar way – even though we had no way of inspecting one in isolation from the other. Don't physicists say something like this about the electrons inside an atom? We can verify *that* there are two, that is to say a certain property of the whole configuration, even though there is no way of detecting any character that uniquely characterises any element of the configuration.

A: But if you were to approach your two stars one would have to be on your left and one on the right'. And this would distinguish them.
B: I agree. Why shouldn't we say that the two stars are distinguishable – meaning that it would be possible for an observer to see one on his left and the other on his right, or more generally, that it would be *possible* for one star to come to have a relation to a third object that the second star would not have to that third object.
A: So you agree with me after all.
B: Not if you mean that the two stars do not have all their properties in common. All I said was that it was logically possible for them to enter into different relationships with a third object. But this would be a change in the universe.
A: If you are right, nothing unobserved would be observable. For the presence of an observer would always change it, and the observation would always be an observation of something else.
B: I don't say that every observation changes what is observed. My point is that there isn't any *being to the right* or *being to the left* in the two-sphere universe until an observer is introduced, that is to say until a real change is made.
A: But the spheres themselves wouldn't have changed.
B: Indeed they would: they would have acquired new relational characteristics. In the absence of any asymmetric observer, I repeat, the spheres would have all their properties in common (including, if you like, the power to enter into different relations with other objects). Hence the principle of identity of indiscernibles is false.
A: So perhaps you really do have twenty hands after all?
B: Not a bit of it. Nothing that I have said prevents me from holding that we can verify *that* there are exactly two. But we could know *that* two things existed without there being any way to distinguish one from the other. The Principle is false.
A: I am not surprised that you ended in this way, since you assumed it in the description of your fantastic 'universe'. Of course, if you began by assuming that the spheres were numerically different though qualitatively alike, you could end by 'proving' what you first assumed.
B: But I wasn't 'proving' anything. I tried to support my contention that it is logically possible for two things to have all their properties in common by giving an illustrative description. (Similarly, if I had to show it is logically possible for nothing at all to be seen, I would ask you to imagine a universe in which everybody was blind.) It was for you to show that my description concealed some hidden contradiction. And you haven't done so.
A: All the same I am not convinced.
B: Well, then, you ought to be.