4.3 Venn Diagrams, part 1

<u>1. Aristotle vs. Boole:</u> Before moving on, it will be helpful to gain some understanding of what "existential import" is.

> **Existential Import:** A statement has existential import if it implies that something exists.

For instance, consider the two sorts of "particular" categorical propositions:

- "Some pizza has pepperoni on it." (Some S are P)
- "Some cookies do not have chocolate chips." (Some S are not P)

The first statement implies that pizza exists (there IS some pizza somewhere, and it has pepperoni on it). The second implies that cookies exist (there ARE some cookies somewhere, and they don't have chocolate chips in them). Because these propositions imply something about what exists, we say that they have "**existential import**".

But, now consider the two sorts of "universal" categorical propositions:

- "All oceans are salty." (All S are P)
- "No rocks are alive." (No S are P)

Do these imply that there ARE some oceans and rocks in existence? There is some disagreement about this. Aristotle (~350 B.C.) thought the answer was clearly **yes**, while George Boole (~1850 A.D.) thought the answer was **no**.

<u>Boole: Universal propositions **do not** imply existence:</u> To understand why Boole thought universal categorical propositions did NOT have existential import, consider these two statements:

- "All unicorns are magical." (All S are P)
- "No hobbits are very tall." (No S are P)

Do these statements imply that there ARE some unicorns and hobbits? Surely not! This fact led Boole to conclude that universal categorical propositions NEVER imply anything about existence. If I tell you that "All oceans are salty", this does not ENTAIL that there ARE any oceans. All it entails is that, IF there are any oceans, then they are salty.

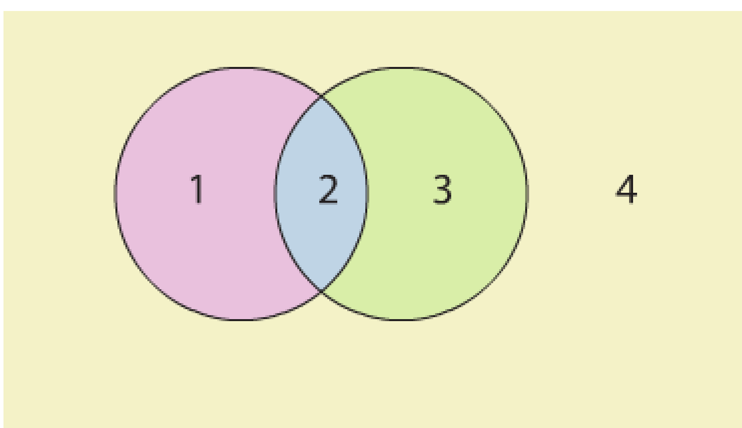
Aristotle: Universal propositions **do** imply existence: Meanwhile, Aristotle thought that they SOMETIMES do imply existence; namely, when the thing mentioned DOES in fact exist.

When I tell you that "All oceans are salty," I AM implying the existence of oceans. But, when I tell you that "All unicorns are magical," I AM NOT implying the existence of oceans. In short, Aristotle believed that universal categorical propositions have existential import whenever they are about things that DO exist, and they do NOT have existential import whenever they are about things that DO NOT exist.

Note: In this course, with a few exceptions, we will assume Boole's interpretation rather than Aristotle's.

2. Venn Diagrams: With Boole's interpretation in mind, let's introduce a handy tool that will help you understand categorical propositions better: Venn Diagrams. There is a way to diagram all four kinds of categorical propositions using two circles. One circle represents the **subject class**, and the other represents the **predicate class**.

Imagine that our **subject class = kittens**, and that our **predicate class = cute**. Below, the mostly pink circle represents the subject class (kittens), so that circle has ONLY kittens in it. Meanwhile, the mostly green circle represents the predicate class (cute things), so that circle has ONLY cute things in it. Finally, the blue-ish place where the two circles overlap (region 2) represents the place where individuals are members of BOTH the subject class AND the predicate class—so region 2 has ONLY cute kittens in it.



Here is what each of the numbered regions would have in them:

1) Region 1 (pink): Kittens that are not cute.
2) Region 2 (blue): Cute kittens.
3) Region 3 (green): Cute things that are not kittens.
4) Region 4 (tan): Things that are neither cute nor kittens.

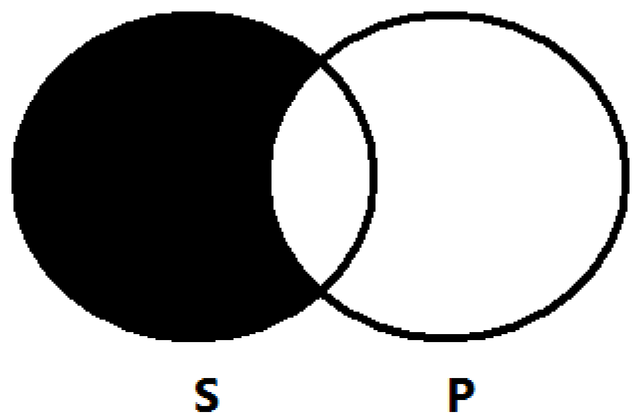
How we will mark Venn diagrams: In this class, a SHADED region will represent a region which has NO members in it. For instance, if there are no such things as **kittens that are not cute**—and let's face it, there probably aren't—then we would **shade** in region 1 to represent the fact that it is empty.

Additionally, an 'X' will represent the fact that there is at least ONE individual in a region. Is there at least one individual that exists in region 3 (cute things that are NOT kittens)? Well, let's brainstorm: Can you think of any **cute things that are NOT kittens**? …Oh! I just thought of one: baby hedgehogs. (If you have any doubts, I dare you to do a google image search for "baby hedgehog"). Since baby hedgehogs are cute things that are not kittens, we know for sure that something exists in region 3. So, we would mark region 3 (the green region) with **an 'X'** to represent the fact that there is something in it.

With that in mind, we would mark the four kinds of categorical proposition as follows:

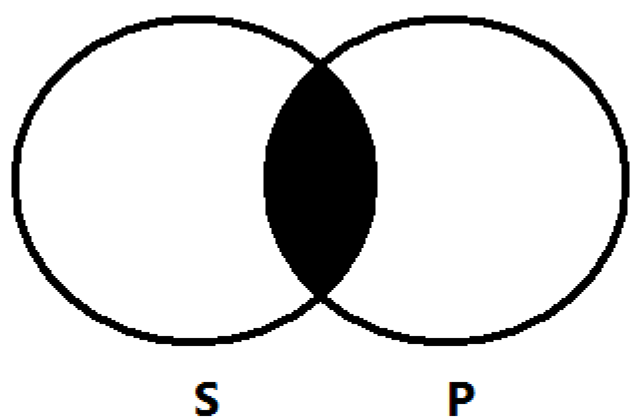
(A) All S are P.

Since all of the S's are P's, there are no S's outside of the P-circle. So, we shade that region to indicate that nothing exists there. However, we do NOT draw an 'X' in the region where S and P overlap; for recall that, on Boole's interpretation, universal statements do NOT imply existence. There may not be any S's at all. All we do know is that, if there ARE some S's, then they are in the region that overlaps with P.



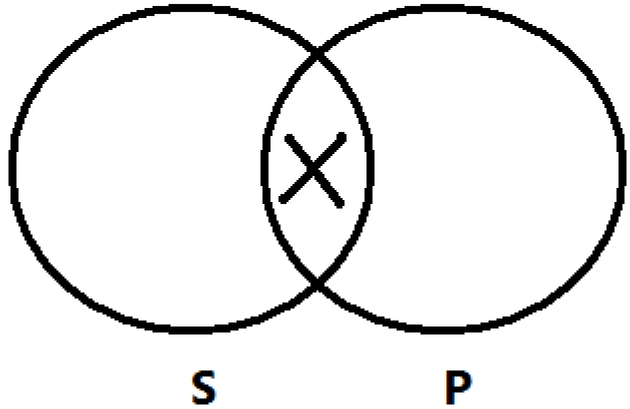
(E) No S are P.

This statement tells us that there are no S's that are also P's. So, we shade in the overlapping region to represent the fact that nothing exists there. But, once again, universal statements do NOT imply that something DOES exist, so we cannot draw an 'X' in any of the unshaded regions. All we know is that, if there ARE some S's, they will be in the part of the S-circle that does not overlap with P; and if there ARE some P's, they will be in the part of the P-circle that does not overlap with S.



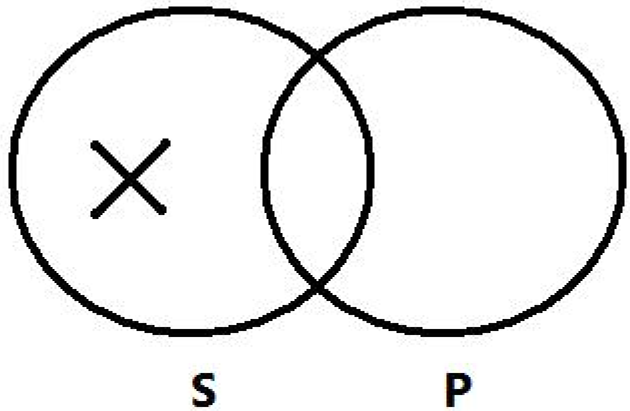
(I) Some S are P.

Particular statements (with the word 'some' as the quantifier) DO have existential import. So, we DO know that some S's exist—and furthermore, the ones that we know to exist are also P's. So, we place an 'X' in the overlapping region in order to represent the fact that something exists there.



(O) Some S are not P.

Again, particular statements DO imply existence. So, we DO know that some S's exist, and the ones that we know to exist are NOT P's. So, we draw an 'X' in the part of the S-circle that does not overlap with the P-circle in order to represent the fact that something exists there.



*Note: **Universal** statements (A and E) always have **shading** on their Venn diagrams, while **particular** statements (O and I) always have an **'X'** on their Venn diagrams.*

<u>3. The Square of Opposition:</u> Now we will introduce another tool that will help you to better understand categorical statements: The square of opposition. The square is based on the fact that each of the four kinds of categorical proposition DIRECTLY CONTRADICTS exactly one of the other four. Here are the two pairs that are in tension with one another:

- (A) "All S are P" **←CONTRADICTS→** (O) "Some S are not P"
- (E) "No S are P" **←CONTRADICTS→** (I) "Some S are P"

*Note: For both pairs of contradictory propositions, (1) One is universal, and the other is particular. (2) Also, **one SHADES** a particular region, while **the other PLACES AN 'X'** there instead.*

To illustrate: Look back at the (A) and (O) diagrams. The (A) diagram shades the leftmost region, indicating that nothing exists there, while the (O) diagram places an 'X' there instead, indicating that something DOES exist there. This is why A-propositions and O-propositions are contradictories. The same applies to the E and I diagrams as well.

(A) and (O) contradict one another: (A) propositions say that, if any S's exist, then ALL of them are P's. But, (O) propositions say just the opposite: They say that some S's DO exist, and they are NOT P's. So these two propositions are in direct conflict with one another. That being the case, we know that whenever one of them is true, we can *immediately* conclude that the other MUST BE false (and vice versa). For instance,

- If "All mice are rodents" is true, then it MUST be the case that "Some mice are not rodents" is false (and vice versa).
- If "All cheese is moldy" is false, then it MUST be the case that "Some cheese is not moldy" is true (and vice versa).

(E) and (I) contradict one another: (E) propositions say that, if any S's exist, then NONE of them are P's. But, (I) propositions say just the opposite: They say that some S's DO exist, and they ARE P's. So these two propositions are in direct conflict with one another. That being the case, we know that whenever one of them is true, we can *immediately* conclude that the other is false (and vice versa). For instance,

- If "No ice cubes are warm" is true, then it MUST be the case that "Some ice cubes are warm" is false (and vice versa).
- If "No comedians are funny" is false, then it MUST be the case that "Some comedians are funny" is true (and vice versa).

Since these sorts of inferences are IMMEDIATE, the following are valid arguments:

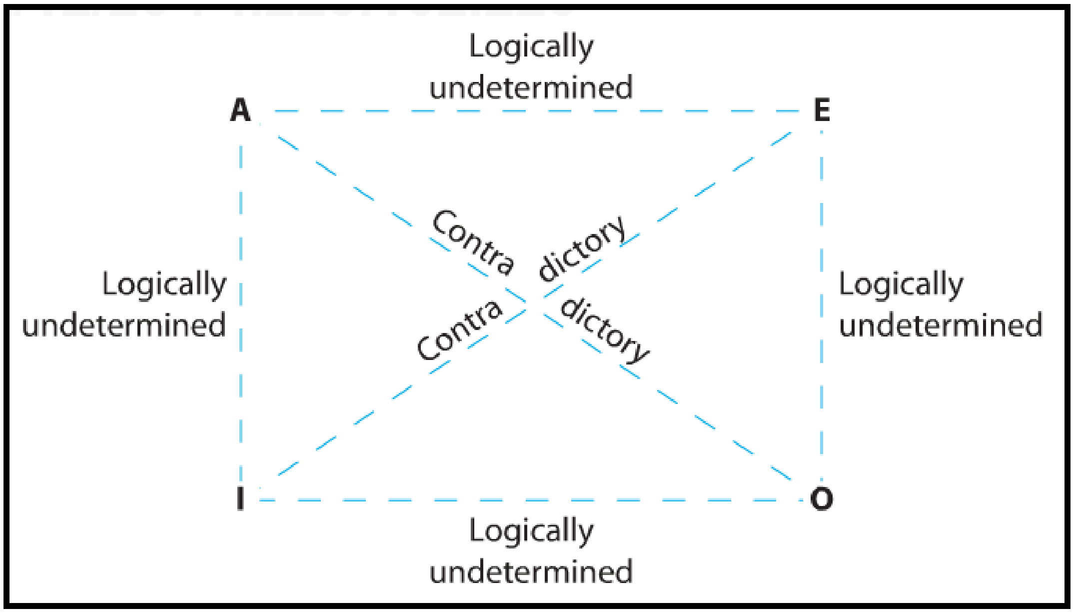
<u>Argument #1:</u>

1. Some cheese is not moldy. ("Some S are P" is true)
2. Therefore, it is NOT the case that all cheese is moldy. (So, "All S are P" is false)

<u>Argument #2:</u>

1. It is NOT the case that some ice cubes are warm. ("Some S are P" is false)
2. Therefore, no ice cubes are warm. (So, "No S are P" is true)

The Boolean Square: It may be helpful to think of this relation as a square, like this:



Above, the propositions that are diagonal from one another contradict each other.

4. The Above Concepts in Practice: Now we will take everything we've learned in this lecture and put it to use, to assess the validity of some arguments. First, two points:

(a) Assessing Validity: To assess the validity of a direct inference (such as Argument #1 and Argument #2 above), we create diagrams for both the premise AND the conclusion, and see if the 'X' or the shading of the conclusion diagram also appears in the premise diagram. If it DOES, then the argument is valid. If it does NOT, then the argument is invalid.

(b) False Propositions: Often, the premise or the conclusion we are given will claim that some statement is false. For instance, the conclusion of Argument #1 above states that "All cheese is moldy" is false (I wrote this as "It is NOT the case that all cheese is moldy" – but this means the same thing). Similarly, the premise of Argument #2 states that "Some ice cubes are warm" is false.

When diagramming a false proposition, (1) First figure out what the diagram of the proposition would be if it was NOT false (that is, if it were TRUE). (2) Next, if the diagram has an 'X', then replace the 'X' with shading. Or, if that diagram has shading, then replace that shading with an 'X'. [This is based on the stuff about contradiction that we learned above]

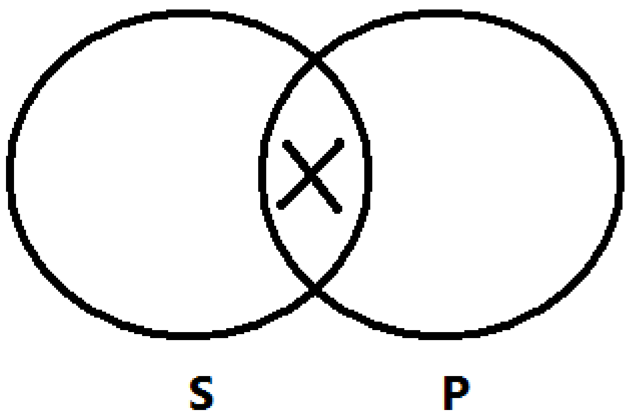
Practicing: Let's try a couple of examples.

<u>Argument #2:</u>

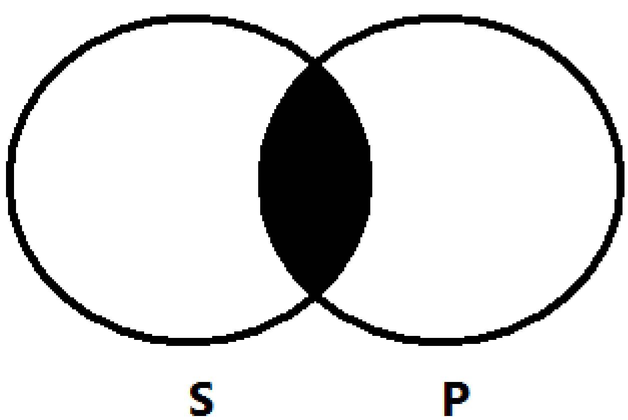
1. It is NOT the case that some ice cubes are warm. ("Some S are P" is false)
2. Therefore, no ice cubes are warm. ("No S are P" is true)

To determine whether this is **valid** or **invalid**, we must diagram both the premise and the conclusion, and see if everything in the conclusion diagram is also contained in the premise diagram.

<u>Diagram of the Premise:</u> First, let's do the premise. The premise states that "Some S are P" is FALSE. Remember, for false propositions, FIRST we have to diagram what it would look like if it were TRUE. Here is what "Some S are P" would look like if it were true:

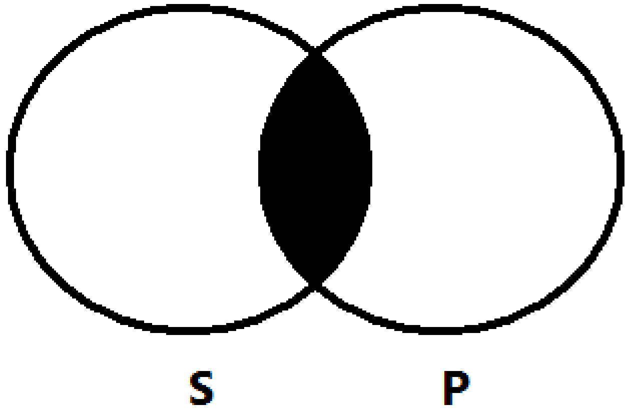


But, since the proposition is FALSE, we replace the 'X' with shading instead. Here is the final diagram for the premise, "It is NOT the case that some ice cubes are warm":



Premise

<u>Diagram of the Conclusion:</u> Now, let's diagram the conclusion. The conclusion states that "No S are P" is true. We know that the diagram for this type of claim looks like this:



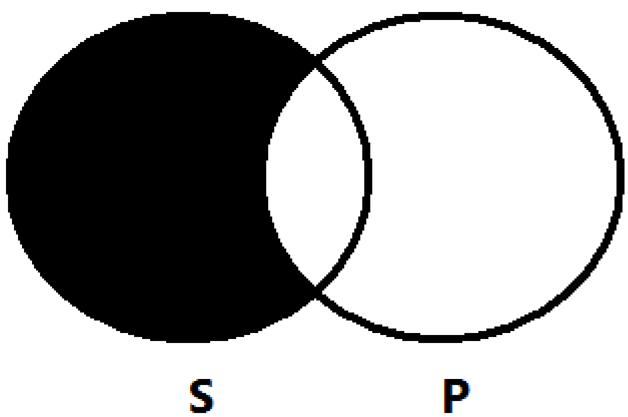
Conclusion

<u>Assessing Validity:</u> Is all of the information from the conclusion diagram (the shaded portion) included in the premise diagram? Sure enough, it IS! The overlapping region is shaded in BOTH diagrams. So, **Argument #2 is valid**. Let's try another one:

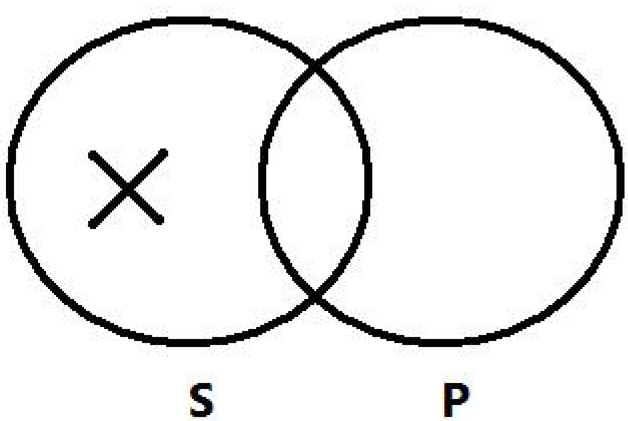
<u>Argument #3:</u>

1. It is NOT the case that all birds can fly. ("All S are P" is false)
2. Therefore, no birds can fly. (So, "No S are P" is true)

<u>Diagram of the Premise:</u> Is this argument valid? Well, let's diagram it. Start with the premise. If the premise were TRUE (i.e., if it just said "All S are P"), it would look like this.

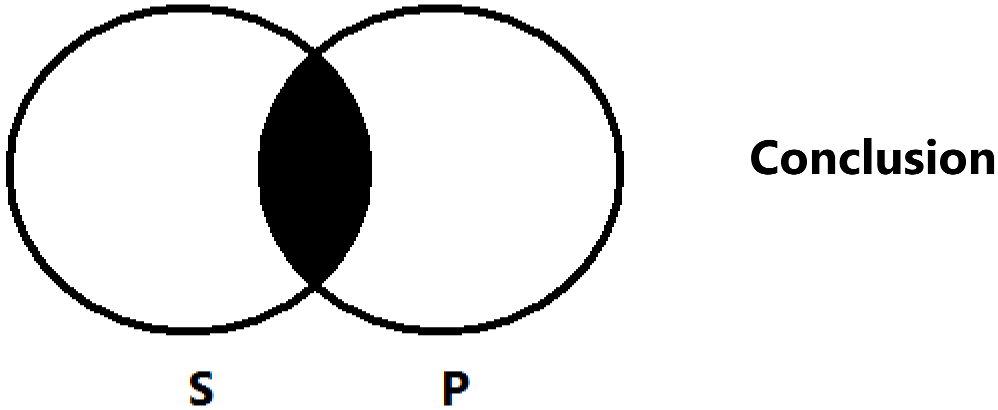


But, since it is FALSE, we should replace the shading with an 'X' instead. Like this:



Premise

Diagram of the Conclusion: Now, let's diagram the conclusion. It states that "No S are P." We know that the graph for THAT type of proposition looks like this:



Assessing Validity: Now, let's compare the graphs for the premise and the conclusion. Is the shaded portion in the conclusion diagram included in the premise diagram? Nope! It sure isn't. So, we know that **Argument #3 is invalid**. Neat trick, right!?

Let's do ONE more… Consider this argument:

Argument #4:

1. All kittens are cute. ("All S are P" is true)
2. Therefore, some kittens are cute. (So, "Some S are P" is true)

Diagram of the Premise: First, we'll diagram the premise. We know that "All S are P" statements look like this:

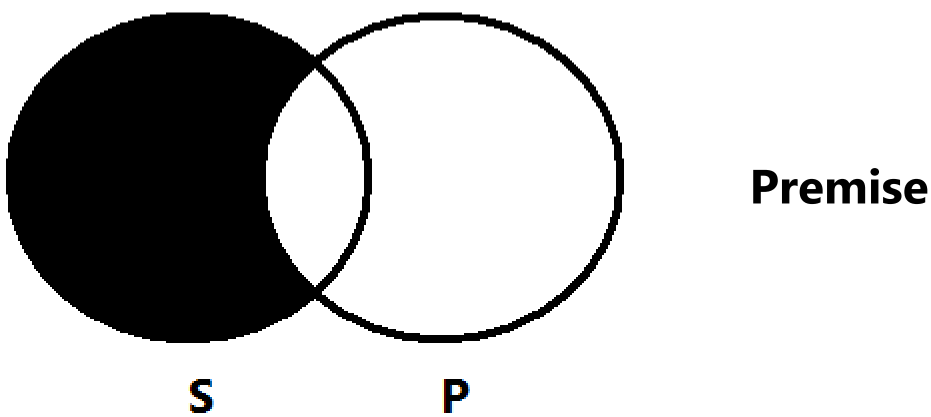
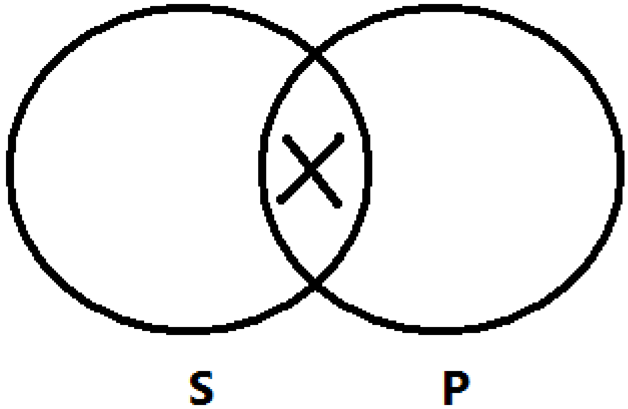


Diagram of the Conclusion: Now, let's do the conclusion. We know that "Some S are P" statements look like this:



Conclusion

Is the ‘X’ from the conclusion diagram included in the premise diagram? NO! So, it looks like **Argument #4 is invalid**. That’s weird though, because it seems like, if we know that ALL kittens are cute, then we should be able to infer that at least SOME are cute… Right?

Answer: NO. Remember what Boole said about existential import. “ALL kittens are cute” is a **universal** proposition—and universal propositions do NOT imply that anything exists. On the other hand, “SOME kittens are cute” is a **particular** proposition—and particular propositions DO imply that something exists. So, we can imagine that the premise only claims that IF there are any kittens, then they are definitely cute. But, maybe there AREN’T any kittens at all! The conclusion, on the other hand, says that some kittens DEFINITELY DO exist, and they are cute.

Existential Fallacy: What has just happened in argument #4 is called an “existential fallacy.” The argument is invalid because it goes from a universal claim (which does NOT entail the existence of anything) and infers a particular claim (which DOES entail the existence of something).

<u>**Note:** The existential fallacy is ONLY committed when:</u> IF the premise DID have existential import, then the argument WOULD be valid. This may seem strange, but to understand why we must treat universal propositions this way, imagine that Argument #4 was about unicorns instead:

1. All unicorns are magical. (“All S are P” is true)
2. Therefore, some unicorns are magical. (So, “Some S are P” is true)

All the premise says is that IF there were any unicorns, then they would all be magical. But, the conclusion says something much stronger: It says that there ARE some unicorns, and they ARE magical! While it would be really great if we could prove the existence of unicorns in this way, unfortunately the argument is invalid because it commits the existential fallacy.

Note: Do homework for section 4.3 at this time.