1.5 Proving Invalidity

1. On Valid Argument Forms: As we have seen, an argument is **valid** if the conclusion is necessarily true whenever the premises are true. But, how can we tell that? It will help to analyze the FORM of various valid and invalid arguments. Consider this argument:

"Dolphins are mammals, so they must have hair at some point."

This is pretty informal. So, first, we have to organize it. We typically put the statements into a numbered list, like this:

1. All dolphins are mammals.
2. Therefore, all dolphins are things that have hair at some point.

Statement 1 is the premise, and statement 2 is the conclusion. But, notice that this argument is not valid as it is stated above. For, the conclusion does not follow from the premises. To make it valid, we'll need to ADD a claim about the relationship between being a mammal and having hair. Something like:

1. All dolphins are mammals.
2. *All mammals are things that have hair at some point.*
3. Therefore, all dolphins are things that have hair at some point.

This is a valid argument. (As it turns out, it is also sound, and dolphins DO in fact have a little bit of bristly hair on their chins when they are first born)

Now, notice the FORM that the argument about dolphins takes:

1. All A are B.
2. All B are C.
3. Therefore, all A are C.

Since the argument was valid, we know that this FORM of argument is valid. And, once we determine the FORM of a valid argument, we can substitute ANYTHING in for the variables (A, B, and C) and it will ALWAYS be a valid argument. Pretty neat, huh? For instance, make A=oceans, B=things that are salty, and C=things with chlorine in them:

1. All oceans are things that are salty.
2. All things that are salty are things that have chlorine in them (salt = NaCl).
3. Therefore, all oceans are things that have chlorine in them.

The argument above (about salt and oceans) was SOUND because its premises were true. But, remember that valid arguments NEED NOT HAVE TRUE PREMISES. The following argument takes the same form, but has totally FALSE premises:

1. All hippos are things that can fly.
2. All things that can fly are things have been to the moon.
3. Therefore, all hippos are things that have been to the moon.

All of the arguments above are valid—for they ALL have the form, "All A are B, and All B are C, therefore all A are C" described above. Remember, validity has nothing to do with the **truth** of the premises or the conclusion—rather, it has only to do with the **form** of the argument.

2. On Invalid Argument Forms: Here is a form of argument that is NOT valid:

1. All people that wear tight jeans are hipsters.
2. All people that collect vinyl are hipsters.
3. Therefore, all people who wear tight jeans are people who collect vinyl.

The form of the argument can be expressed as follows:

1. All A are B.
2. All C are B.
3. Therefore, all A are C.

To see that this is not valid, we only need to come up with one example where the premises are true, but the conclusion is clearly false. Consider:

1. All birds are things that can fly.
2. All airplanes are things that can fly.
3. Therefore, all birds are airplanes.

Hooray! We have discovered an invalid argument form by finding what is called a "counter-example." We can use this method to detect invalid arguments:

> **The Counter-Example Method:** Once you determine the form that an argument has, if you can find an example of an argument with that form where the premises are true and the conclusion is false, then that argument form is invalid.

In other words, if we can come up with just ONE example of an argument form where the premises are true and the conclusion is false, we know that ALL arguments of that form are invalid.

Why does this method work? Well, remember that, if an argument is valid, then there is NO WAY that the premises can be true and the conclusion false at the same time. So, if you DO find a scenario where that happens (true premises and a false conclusion), then you know for sure that the argument form is NOT valid. Let's do another one:
Imagine that someone told you that, "Some of the people who oppose abortion are conservatives, and some of the people who oppose same-sex marriage are conservatives. So, there must be some people who oppose both abortion AND same-sex marriage."

Ok, first, let's write that down in an organized, numbered way:

1. Some people who oppose abortion are conservatives.
2. Some people who oppose same-sex marriage are conservatives.
3. Therefore, some people who oppose abortion are people who oppose same-sex marriage.

As it turns out, the premises AND the conclusion here are all true. But, the inference is nevertheless invalid. Let's write down the FORM of the argument. Let A="people who oppose abortion", let B="conservatives", and let C="people who oppose same-sex marriage". Now, the form of the argument is as follows:

1. Some A are C.
2. Some B are C.
3. Therefore, some A are B.

We can easily come up with a counter-example to show that this argument form is an invalid one; i.e., here is an argument with the same form, but with true premises and a false conclusion:

1. Some male students are logic students.
2. Some female students are logic students.
3. Therefore, some male students are female students.

This argument has true premises and a false conclusion, but has exactly the same FORM as the argument above about conservatives. Therefore, the argument about conservatives is invalid.

One more. Someone says, "Some fast-food employees are not well-paid, and all KFC employees are fast-food employees. So, some KFC employees are not well-paid." Let A="Fast-food employees", let B="people who well-paid", and let C="KFC employees". Here is the argument's form:

1. Some A are not B.
2. All C are A.
3. Therefore, some C are not B.

But, here is an argument with the same form, where the premises are clearly true and the conclusion is clearly false:

1. Some mammals are not human beings.
2. All CU students are mammals.
3. Therefore, some CU students are not human beings.

Eureka! We found another invalid argument form. Let's do one more: "If you had taken Logic, you would be rational. But, you didn't, so you're irrational." Now, let's write it more clearly:

1. If you have completed Logic, then you are rational.
2. You have not completed Logic.
3. Therefore, you are not rational.

That doesn't seem right. Surely you might already be rational BEFORE completing Logic, right? So, this must be an invalid argument form. Let A="you have completed Logic", and B="you are rational". The form of the argument is:

1. If A, then B.
2. Not A (or, in other words, "A is false")
3. Therefore, Not B.

Here is a counter-example:

1. If Benji is a cat, then Benji is a mammal.
2. Benji is not a cat.
3. Therefore, Benji is not a mammal.

You'll see people use this form all the time, and we'll study it more later in the semester).

Note: Do homework for section 1.5 at this time.