6.3 Truth Tables for Propositions

1. Truth Tables for 2-Letter Arguments: In section 6.3, we learned how to make truth tables for propositions. For instance, the truth table for "W ≡ (B • T)" is determined in the following way, where the final results are in red:

W ≡ (B • T)
T **T** T **T** T
T **F** T **F** F
T **F** F **F** T
T **F** F **F** F
F **F** T **T** T
F **T** T **F** F
F **T** F **F** T
F **T** F **F** F

The resulting truth table for the given proposition looks like this:

W	**B**	**T**	**W ≡ (B • T)**
T	T	T	**T**
T	T	F	**F**
T	F	T	**F**
T	F	F	**F**
F	T	T	**F**
F	T	F	**T**
F	F	T	**T**
F	F	F	**T**

In this section, we will learn how to make truth tables for arguments, composed of TWO or more propositions. Let's start with a statement which has ONLY TWO propositions as components (one as a premise and one as a conclusion).

Example #1: For instance, imagine that I told you: "If you don't study, then you will not get good grades. Therefore, if you study, then you will get good grades." Let "S"="You study" and "G"="You get good grades". In that case, my argument can be written as the following:

Premise: ~S Ɔ ~G
Conclusion: S Ɔ G

Is this argument **valid** or **invalid**? In order to answer that question, we will need to draw up truth tables for BOTH the premise AND the conclusion. Before we start, we first write up a four line truth table, with truth values for S and G, like this:

S	**G**	**~S Ↄ ~G**		**S Ↄ G**
T	T	?		?
T	F	?		?
F	T	?		?
F	F	?		?

The shading represents the divide between the premise and the conclusion. Recall back to section 1.4, when we defined the term "valid". There, we said that a valid argument is one for which **it is impossible for the premises to be true and the conclusion false**.

In section 1.5, we used "The counter-example method" to determine whether or not an argument is invalid. If we could come up with an argument with the same FORM, where the premises were obviously true and the conclusion was obviously false, then we concluded that the argument was INVALID.

That is basically what we'll be doing here, but with truth tables. We will write up truth tables for the premises and the conclusion, and **if there is any line where the premises are ALL true, and the conclusion is false, then the argument is invalid. Otherwise (if there is no such line where this occurs), the argument is valid.**

Let's do the truth table for the argument above now. First, we simply fill in the truth values under S and G, like this:

S	**G**	**~S Ↄ ~G**		**S Ↄ G**
T	T	T T		T T
T	F	T F		T F
F	T	F T		F T
F	F	F F		F F

Next, let's get rid of the negations in the premise, like this:

S	**G**	**~S Ↄ ~G**		**S Ↄ G**
T	T	**F**T **F**T		T T
T	F	**F**T **T**F		T F
F	T	**T**F **F**T		F T
F	F	**T**F **T**F		F F

Finally, we can solve for the conditionals ("Ɔ"). Remember that a conditional is ALWAYS true unless it has a **true antecedent and a false consequent**.

S	**G**	**~S Ɔ ~G**		**S Ɔ G**
T	T	**F**T **T** **F**T		T **T** T
T	F	**F**T **T** **T**F		T **F** F
F	T	**T**F **F** **F**T		F **T** T
F	F	**T**F **T** **T**F		F **T** F

Now, in order to determine whether this argument is valid or invalid, we simply look for a line where the premise(s) are true and the conclusion is false. Look at each line. Is there any line where the red letter on the left is a "**T**" and the letter on the right is an "**F**"? There sure is! The second line does this:

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S	**G**	**~S Ɔ ~G**		**S Ɔ G**
T	T	**F**T **T** **F**T		T **T** T
T	F	**F**T **T** **T**F		T **F** F
F	T	**T**F **F** **F**T		F **T** T
F	F	**T**F **T** **T**F		F **T** F

Because there IS a line where the premise is true and the conclusion is false (circled in red above), **this argument is INVALID**. In other words, even if it is true that those who don't study will get bad grades, this does NOT entail that those who DO study WILL get GOOD grades. The inference is invalid.

1. Truth Tables for 3-Letter Arguments: Let's try a more complicated argument with MULTIPLE premises and THREE different statement letters:

Example #2: Let's do #10 from your textbook from exercise 6.4, section I (page 348). It says, "If racial quotas are adopted for promoting employees, then qualified employees will be passed over; but if racial quotas are not adopted, then prior discrimination will go unaddressed. Either racial quotas will or will not be adopted for promoting employees. Therefore, either qualified employees will be passed over or prior discrimination will go unaddressed."

Let "R"="Racial quotas are adopted", "Q"="Qualified employees are passed over", and "P"="Prior discrimination goes unaddressed". In that case, the argument can be written as the following:

1. R Ɔ Q
2. ~R Ɔ P
3. R ∨ ~R
4. Therefore: Q ∨ P

The truth table to be filled in will look like the following:

R	P	Q	R Ɔ Q	~R Ɔ P	R ∨ ~R		Q ∨ P
T	T	T	?	?	?		?
T	T	F	?	?	?		?
T	F	T	?	?	?		?
T	F	F	?	?	?		?
F	T	T	?	?	?		?
F	T	F	?	?	?		?
F	F	T	?	?	?		?
F	F	F	?	?	?		?

Let's start by filling in all of the truth values for R, P, and Q, like this:

R	P	Q	R Ɔ Q	~R Ɔ P	R ∨ ~R		Q ∨ P
T	T	T	T T	T T	T T		T T
T	T	F	T F	T T	T T		F T
T	F	T	T T	T F	T T		T F
T	F	F	T F	T F	T T		F F
F	T	T	F T	F T	F F		T T
F	T	F	F F	F T	F F		F T
F	F	T	F T	F F	F F		T F
F	F	F	F F	F F	F F		F F

Next, let's get rid of the negations in the second and third premise, like this:

R	P	Q	R Ɔ Q	~R Ɔ P	R ∨ ~R		Q ∨ P
T	T	T	T T	**F**T T	T **F**T		T T
T	T	F	T F	**F**T T	T **F**T		F T
T	F	T	T T	**F**T F	T **F**T		T F
T	F	F	T F	**F**T F	T **F**T		F F
F	T	T	F T	**T**F T	F **T**F		T T
F	T	F	F F	**T**F T	F **T**F		F T
F	F	T	F T	**T**F F	F **T**F		T F
F	F	F	F F	**T**F F	F **T**F		F F

Next, we can solve for all of the premises and the conclusion. The first two premises are **conditionals** (which are ONLY false when they have a true antecedent and a false consequent), while the third premise and the conclusion are **disjunctions** (which are ONLY false when BOTH disjuncts are false). Here is the result:

R	P	Q	R Ↄ Q	~R Ↄ P	R v ~R		Q v P
T	T	T	T **T** T	FT **T** T	T **T** FT		T **T** T
T	T	F	T **F** F	FT **T** T	T **T** FT		F **T** T
T	F	T	T **T** T	FT **T** F	T **T** FT		T **T** F
T	F	F	T **F** F	FT **T** F	T **T** FT		F **F** F
F	T	T	F **T** T	TF **T** T	F **T** TF		T **T** T
F	T	F	F **T** F	TF **T** T	F **T** TF		F **T** T
F	F	T	F **T** T	TF **F** F	F **T** TF		T **T** F
F	F	F	F **T** F	TF **F** F	F **T** TF		F **F** F

Now we look for lines where the conclusion is false and the premises are true. There are only 2 lines where the conclusion is false, so we should only be concerned with those:

R	P	Q	R Ↄ Q	~R Ↄ P	R v ~R		Q v P
T	T	T	T **T** T	FT **T** T	T **T** FT		T **T** T
T	T	F	T **F** F	FT **T** T	T **T** FT		F **T** T
T	F	T	T **T** T	FT **T** F	T **T** FT		T **T** F
T	F	F	T **F** F	FT **T** F	T **T** FT		F **F** F
F	T	T	F **T** T	TF **T** T	F **T** TF		T **T** T
F	T	F	F **T** F	TF **T** T	F **T** TF		F **T** T
F	F	T	F **T** T	TF **F** F	F **T** TF		T **T** F
F	F	F	F **T** F	TF **F** F	F **T** TF		F **F** F

The two yellow highlighted lines are the ONLY lines where the conclusion is false. We should ignore all of the others. Now ask, are either of those lines ones where ALL of the premises are true?

Nope… On the first yellow line, premise 1 (R Ↄ Q) is false. On the second yellow line, the second premise (~R Ↄ P) is false. So, since there ARE NOT any lines where all of the premises are true and the conclusion is false, **the argument is valid**.

Note: Do homework for section 6.4 at this time.