Proving Invalidity Using Counterexamples

<u>1. On Valid Argument Forms:</u> As we have seen, an argument is **valid** if the conclusion is necessarily true whenever the premises are true. But, how can we tell that? It will help to analyze the FORM of various valid and invalid arguments. Consider this argument:

> "Dolphins are mammals, so they must have hair."

This is pretty informal. So, first, we have to organize it. We typically put the statements into a numbered list, like this:

1. All dolphins are mammals.
2. Therefore, all dolphins are things that have hair.

Statement 1 is the premise, and statement 2 is the conclusion. But, notice that this argument is not valid as it is stated above. For, the conclusion does not follow from the premises. To make it valid, we'll need to ADD a claim about the relationship between being a mammal and having hair. Something like:

1. All dolphins are mammals.
2. *All mammals are things that have hair.*
3. Therefore, all dolphins are things that have hair.

This is a valid argument. (As it turns out, it is also sound, and dolphins DO in fact have a little bit of bristly hair on their chins—but only when they are first born)

Now, notice the FORM that the argument about dolphins takes:

1. All A's are B's.
2. All B's are C's.
3. Therefore, all A's are C's.

Since the argument was valid, we know that this FORM of argument is valid. And, once we determine the FORM of a valid argument, we can substitute ANYTHING in for the variables (A, B, and C) and it will ALWAYS be a valid argument. Pretty neat, huh? For instance, make A=oceans, B=things that are salty, and C=things with chlorine in them:

1. All oceans are (things that are) full of salt.
2. All things that are full of salt are things that are full of Chlorine (salt = NaCl).
3. Therefore, all oceans are things that are full of Chlorine.

The argument above (about salt and oceans) was SOUND because, as it turns out, its premises were true. But, remember that valid arguments NEED NOT HAVE TRUE PREMISES. The following argument takes the same form, but has totally FALSE premises:

1. All hippos are things that can fly.
2. All things that can fly are things have been to the moon.
3. Therefore, all hippos are things that have been to the moon.

All of the arguments above are valid—for they ALL have the form, "All A's are B's, and All B's are C's, therefore all A's are C's" described above. Remember, validity has nothing to do with the **truth** of the premises or the conclusion—rather, it has only to do with the **form** of the argument.

2. On Invalid Argument Forms: Here is a form of argument that is NOT valid:

1. All people who wear 3-cornered hats are Colonial Williamsburg employees.
2. All people who fire canons daily are Colonial Williamsburg employees.
3. Therefore, all people who wear 3-cornered hats are people who fire canons daily.

The form of the argument can be expressed as follows:

1. All A's are B's.
2. All C's are B's.
3. Therefore, all A's are C's.

To see that this is not valid, we only need to come up with one example where the premises are true, but the conclusion is clearly false. Consider:

1. All dogs are mammals.
2. All cats are mammals.
3. Therefore, all dogs are cats.

Hooray! We have discovered an invalid argument form by finding what is called a "counter-example." We can use this method to detect invalid arguments:

> **The Counter-Example Method:** Once you determine the form that an argument has, if you can find an example of an argument with that same form where the premises are true and the conclusion is false, then that argument form is invalid.

In other words, if we can come up with just ONE example of an argument form where the premises are true and the conclusion is false, we know that ALL arguments of that form are invalid.

Why does this method work? Well, remember that, if an argument is valid, then there is NO WAY that the premises can be true and the conclusion false at the same time. So, if you DO find a scenario where that happens (true premises and a false conclusion), then you know for sure that the argument form is NOT valid. Let's do another one:

> A friend says: "Some of the people who favor legalized abortion are liberals, and some of the people who favor stricter gun regulations are liberals. So, there must be some people who favor both legalized abortion AND stricter gun regulations."

Ok, first, let's write that down in an organized, numbered way:

1. Some people who favor legalized abortion are liberals.
2. Some people who favor stricter gun regulations are liberals.
3. Therefore, some people who favor legalized abortion are people who favor stricter gun regulations.

As it turns out, the premises AND the conclusion here are all true. But, the inference is nevertheless invalid. Let's write down the FORM of the argument. Let A="people who favor legalized abortion", let B="liberals", and let C="people who favor stricter gun regulations". Now, the form of the argument is as follows:

1. Some A's are B's.
2. Some C's are B's.
3. Therefore, some A's are C's.

We can easily come up with a counter-example to show that this argument form is an invalid one; i.e., here is an argument with the same form, but with true premises and a false conclusion:

1. Some people in this class are people who have attended W&M.
2. Some former U.S. presidents are people who have attended W&M.
3. Therefore, some people in this room are former U.S. presidents.

This argument has true premises and a false conclusion, but has exactly the same FORM as the argument above about liberals. Therefore, the argument about liberals is invalid. Let's do another. Imagine that someone says,

> "Some fast-food employees are not well-paid, and all Taco Bell employees are fast-food employees. So, some Taco Bell employees are not well-paid."

Let A="Fast-food employees", let B="people who well-paid", and let C="Taco Bell employees". Here is the argument's form:

1. Some A's are not B's.
2. All C's are A's.
3. Therefore, some C's are not B's.

But, here is an argument with the same form, where the premises are clearly true and the conclusion is clearly false:

1. Some mammals are not human beings.
2. All W&M students are mammals.
3. Therefore, some W&M students are not human beings.

Eureka! We found another invalid argument form. Let's do one more:

"If you've passed Logic, you are rational. But, you haven't, so you're irrational."

Now, let's write it more clearly:

1. If you have passed Logic, then you are rational.
2. You have not passed Logic.
3. Therefore, you are not rational.

That doesn't seem right. Surely you might already be rational BEFORE completing Logic, right? So, this must be an invalid argument form. Let A="you have completed Logic", and B="you are rational". The form of the argument is:

1. If A, then B.
2. Not A (or, in other words, "A is false")
3. Therefore, Not B.

Here is a counter-example:

1. If Thomas Jefferson were trampled by unicorns, then he would be dead.
2. But, Thomas Jefferson was not trampled by unicorns.
3. Therefore, Thomas Jefferson is not dead.

The premises are true, but the conclusion is clearly false. However, people make this sort of mistake with their inferences all the time (and we'll study it in more detail later).